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Orbital Characteristics of Binary Systems after Asymmetric Supernova Explosions

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ABSTRACT

We present an analytical method for studying the changes of the orbital characteristics of binary systems with circular orbits due to a kick velocity imparted to the newborn neutron star during a supernova explosion (SN). Assuming a Maxwellian distribution of kick velocities we derive analytical expressions for the distribution functions of orbital separations and eccentricities immediately after the explosion, of orbital separations after circularization of the post-SN orbits, and of systemic velocities of binaries that remain bound after the explosion. These distributions of binary characteristics can be used to perform analytical population synthesis calculations of various types of binaries, the formation of which involves a supernova explosion. We study in detail the dependence of the derived distributions on the kick velocity and the pre-SN characteristics, we identify all the limits imposed on the post-SN orbital characteristics, and we discuss their implications for the population of X-ray binaries and double neutron star systems. We show that large kick velocities do not necessarily result in large systemic velocities; for typical X-ray binary progenitors the maximum post-SN systemic velocity is comparable to the relative orbital velocity prior to the explosion. We also find that, unless accretion-induced collapse is a viable formation channel, X-ray binaries in globular clusters have most probably been formed by stellar dynamical interactions only, and not directly from primordial binaries.

Subject headings: stars: binaries – stars: neutron – stars: supernovae – X-rays: binaries

1.   INTRODUCTION

Studies of the radio pulsar population (e.g., Gunn & Ostriker 1970; Helfand & Tademaru 1977; Harrison, Lyne & Anderson 1993; Lyne & Lorimer 1994) have shown that pulsars move in the Galaxy with very high space velocities, ranging from 20 to $2000\,\mathrm{km\,s^{-1}}$, and that their galactic distribution has a large scale height, of the order of $1\,\mathrm{kpc}$. The origin of these high velocities is often attributed to a kick velocity imparted to the neutron star at the time of the supernova explosion. Early studies by Dewey & Cordes (1987) and Bailes (1989) concluded that the mean magnitude of the kick velocity is of the order of $100-200\,\mathrm{km\,s^{-1}}$. In a more recent study, which takes into account new measurements of pulsar proper motions, a new electron density model, and a selection effect against fast pulsars, Lyne & Lorimer (1994) found the mean kick velocity to be $\sim 450 \pm 90\,\mathrm{km\,s^{-1}}$. Additional observational evidence in support of a kick velocity imparted to neutron stars at birth are related to the existence of a high–velocity population of $\mathrm{O,B}$ runaway stars (e.g., Stone 1991), as well as to supernova remnant – pulsar associations, studies of which yield kick velocities up to $2000\,\mathrm{km\,s^{-1}}$ (Caraveo 1993; Frail, Goss, & Whiteoak 1994).

In contrast to Dewey & Cordes (1987), Iben & Tutukov (1996) have recently concluded that the hypothesis of natal kicks imparted to neutron stars is unnecessary. They have found that the transverse velocity distribution of pulsars in the solar neighborhood, as well as that of $\mathrm{O,B}$ runaway stars, massive X-ray binaries, and double neutron stars, can be explained by the recoil velocity due to symmetric supernova explosions. However, they reach this conclusion by assuming (i) that all stars are members of binary systems and (ii) that neutron stars formed by massive single stars (formed only by mergers) or in wide binary systems rotate too slowly to become radio pulsars. Although, their results are marginally consistent (mean predicted velocities are $\sim 100-150\,\mathrm{km\,s^{-1}}$) with the old pulsar distance scale (Harrison et al. 1993), they are not consistent with the more recent results of Lyne & Lorimer (1994).

Over the years, several theories have been put forward in an effort to explain the origin of kick velocities (e.g., Harrison & Tademaru 1975; Chugai 1984; Duncan & Thomson 1992; Herant, Benz & Colgate 1992; Janka & Müller 1994; Burrows, Hayes, & Fryxell 1995; Burrows & Hayes 1996). Even a small asymmetry during the collapse of the core can give a kick to the remnant of the explosion. The asymmetry may be related either to neutrino emission or to mass ejection during the supernova, and may be caused by the magnetic field or rotation of the collapsing core, or by hydrodynamic instabilities, such as Rayleigh-Taylor or convective motions. In any case, the mechanism responsible for the kick velocity is still not well understood, and it appears that fully three-dimensional numerical simulations of the core collapse will be required in order to settle this issue.

Several authors have previously studied the effect of an asymmetric supernova explosion on binary parameters, focusing on various aspects of the problem. Early work by Flannery & van den Heuvel (1975), Mitalas (1976), Sutantyo (1978), and Hills (1983) addressed the problem of deriving expressions of post-SN orbital characteristics for a specific kick velocity for both circular and eccentric pre-SN orbits. They also derived survival probabilities for kick velocities of constant magnitude and random direction. The one-to-one link between pre-SN and post-SN parameters is broken when kick velocities are allowed to have a distribution over both magnitude and direction, in which case there exists a distribution of post-SN characteristics, even for pre-SN binaries with specific orbital parameters. Wijers, van Paradijs, & van den Heuvel (1992) were the first to address this problem. They derived an analytic expression for the distribution of post-SN orbital separations and eccentricities only, which however was also convolved with a distribution of pre-SN orbital separations. More recently Brandt & Podsiadlowski (1995) addressed the same problem using numerical methods (Monte Carlo simulations). The resulting distributions are again convolved with pre-SN period distributions, and, in this case, are calculated only for specific stellar masses, in an effort to compare them with observation. Because these distributions are calculated numerically, information about the allowed ranges of post-SN characteristics, the shape of multi-dimensional distributions, and their dependence on the pre-SN and kick-velocity characteristics is limited. Our purpose here is to derive analytical expressions of various post-SN characteristics for the realistic case of kick velocities with a distribution in both magnitude and direction. The derived distributions are general, and apply to any circular binary systems that experience asymmetric supernova explosions.

The study presented in this paper has been motivated by our interest in performing population synthesis calculations for low-mass X-ray binaries. Monte Carlo techniques have been widely used in such calculations modeling various kinds of binary systems (e.g., Dewey & Cordes 1987; de Kool 1992; Romani 1992). Another method is based in creating a multi-dimensional grid of initial binary parameters and tracing the evolution of systems through a sequence of evolutionary stages for each set of initial parameters (Kolb 1993; Iben, Tutukov, & Yungel'son 1995 and references therein). Both of these numerical methods have the same problem: although the goal is to calculate the characteristics of the final population, the sampling procedure is applied on the initial population, and therefore it is possible that the final population is under-sampled, even if the sampling of the primordial population appears adequate. Another problem with both methods is related to statistical accuracy: the initial sets of parameters cover a wide range, of which only a small part is populated by progenitors of interest, especially in the case of X-ray binaries, which have very small birth rates; therefore, with these methods it is necessary to study a very large number of primordial binaries in order for a statistically significant

number of systems to survive. Both of these problems are absent in population synthesis calculations performed analytically, where distributions of primordial binaries over orbital characteristics are transformed through a sequence of evolutionary stages, using Jacobian transformations. In this way, the regions in parameter space populated by the progenitors of interest are identified and the final population is calculated directly. This method has been formulated and applied to the study of cataclysmic binaries by Politano (1996) (see also Politano, Ritter, & Webbink 1989; Politano & Webbink 1990). Apart from the absence of the problems discussed above, the analytical method has the additional advantage that the shape of final distributions is calculated exactly, revealing fine details and subtle features, such as sharp peaks, infinities, or definite limits imposed on the final parameters. Also, the various dependences of these parameters and their distributions on the initial parameters can be identified and studied in detail.

In order to use the analytical method in population synthesis of neutron-star binaries, it is necessary to develop an analytical tool for the modeling of asymmetric supernova explosions. Our purpose in this paper is to present such a method based on Jacobian transformations for computing analytically the probability distributions of several orbital characteristics of post-SN binaries. These distributions include (§ 2) the orbital separations and eccentricities immediately after the supernova explosion, the circularized orbital separations, and (§ 3) the systemic velocities. They are derived for circular pre-SN orbits and for kick velocities that are randomly distributed not only in direction but also in magnitude (Maxwellian distribution). The derived expressions can be used in synthesis calculations of any kind of binaries that experience supernova explosions during their evolution. The analytical character of the derivation enables us to perform detailed parameter studies and identify those characteristics of the kick velocities or the pre-SN binaries that govern the behavior of the post-SN distribution functions. Expressions for the limiting cases of very large or very small kick velocities relative to the pre-SN orbital velocities are also derived. We identify the limits imposed on the post-SN parameters and discuss their physical interpretation. In addition, we calculate survival probabilities (§ 4) as functions of the pre-supernova (pre-SN) orbital characteristics and the mean kick velocity. Finally, we examine several implications of our results (§ 5) for the progenitors of high- and low-mass X-ray binaries, double neutron stars, and their populations in globular clusters. A list of the symbols used throughout the paper is given in Appendix A. The study of a few special cases is included in Appendices B and C.

2. POST-SUPERNOVA ORBITS

We assume that the binary orbits prior to the supernova explosion are circular, and that the kick velocities follow a Maxwellian distribution. The first of these two assumptions is unlikely to be violated in the case of systems that have emerged from a common-envelope phase, or have experienced (semi-)conservative mass transfer. Orbital eccentricities may be important for binaries with components that have not interacted prior to the supernova explosion, although it is still possible that circularization has occurred during their main-sequence evolution (see Portegies-Zwart & Verbunt 1996). The second assumption, that of a Maxwellian distribution of kick velocities, we adopt in the absence of an adequate theoretical understanding of their origin. It is conceivable that the direction of the kick is affected by the kinematical or rotational properties of the collapsing core, but any correlation between the kick direction and the orbital rotational axis or the orbital velocity has yet to be established. In principle, a method like that described here can be used with any distribution of kick velocities, although the derivational details will be different.

In most of our calculations the interaction between the expanding supernova shell and the companion to the exploding star has been ignored. According to Fryxell & Arnett (1981), this interaction is generally weak, especially in the case of low-mass X-ray binaries (LMXBs) and double neutron stars, where the solid angle intercepted by the companion is very small unless the orbital separation is also small (see Appendix A and Romani 1992). This effect may be more important for high-mass X-ray binaries (HMXBs), although their orbits are much wider. Nevertheless, for completeness we have repeated the calculation of the probability distribution of circularized orbital separations, including also the effect of the impulse velocity (see Appendix A).

2.1. Non-Circularized Orbits

In this section we derive the distribution of post-SN binary systems over orbital separations and eccentricities immediately after the supernova explosion, by transforming the distribution of kick velocities imparted to the neutron star into a distribution over other binary parameters of interest.

We adopt a reference frame centered on the exploding star mass, M_1, just prior to the supernova. The companion mass, M_2, is chosen to be at rest and the exploding star to move in a circular orbit with separation A_i. The x-axis lies along the line connecting the centers of mass of the two stars, pointing from M_2 to M_1. The y-axis lies parallel to the direction of the pre-SN orbital velocity V_r of M_1 relative to its companion. The z-axis completes a

right-handed orthogonal system (Figure 1).

Two of the parameters characterizing the post-SN binaries are the orbital separation, A_f, and the eccentricity, e. We use the energy and angular momentum equations for eccentric orbits to relate these two parameters to the three components of the kick velocity. The supernova explosion, mass loss, and kick are assumed to be instantaneous. In general, for two stars with masses M_a and M_b in an orbit with orbital separation A and eccentricity e, their relative velocity V at a distance r is given by:

$$V^2 = G\,(M_a + M_b)\,\left(\frac{2}{r} - \frac{1}{A}\right),\qquad(1)$$

and the specific angular momentum of the system is:

$$|\vec{r} \times \vec{V}|^2 = G\,(M_a + M_b)\,A\,(1 - e^2).\qquad(2)$$

We can also apply the above two equations in the case of the post-SN binary, for which $\vec{V} = (V_{kx}, V_{ky} + V_r, V_{kz})$ and $\vec{r} = (A_i, 0, 0)$. The orbital separation, A_f, and the eccentricity, e, of the post-SN orbit are thus related to the components of the kick velocity, V_{kx}, V_{ky}, and V_{kz} by the expressions:

$$A_f = G(M_{NS} + M_2)\left[\frac{2G(M_{NS} + M_2)}{A_i} - V_k^2 - V_r^2 - 2V_{ky}V_r\right]^{-1}\qquad(3)$$

$$1 - e^2 = \frac{(V_{kz}^2 + V_{ky}^2 + V_r^2 + 2V_{ky}V_r)A_i^2}{G(M_{NS} + M_2)A_f},\qquad(4)$$

where M_{NS} is the gravitational mass of the neutron star and V_k is the magnitude of the kick velocity.

The third independent parameter describing the post-SN state of the binary is the orientation of the eccentric orbit relative to the pre-SN orbital plane. The plane of the binary orbit is altered due to the z-component of the kick velocity. Since the explosion is assumed to be instantaneous, the position of the two stars just before and just after the supernova remains unchanged. In our reference frame (Figure 1) the two stars lie along the x-axis, and therefore the intersection of the two orbital planes must coincide with the x-axis. The angle, θ, between the pre- and post-SN orbital planes, is equal to the one between the relative velocity just before the explosion, $\vec{V_r} = (0, V_r, 0)$, and the projection of the relative velocity just after the explosion onto the $y - z$ plane, $\vec{V}_{yz} = (0, V_{ky} + V_y, V_{kz})$. Hence:

$$cos\theta = \frac{\vec{V}_{yz} \cdot \vec{V}_r}{|\vec{V}_r|\,|\vec{V}_{yz}|} = \frac{V_{ky} + V_r}{[(V_{ky} + V_r)^2 + V_{kz}^2]^{1/2}}.\qquad(5)$$

For convenience, we rewrite equations $(3), (4), (5)$ in dimensionless form using the following definitions:

$$\alpha \equiv \frac{A_f}{A_i}, \tag{6}$$

$$\beta \equiv \frac{M_{NS} + M_2}{M_1 + M_2}, \tag{7}$$

$$v_{kj} \equiv \frac{V_{kj}}{V_r}, \tag{8}$$

where j can be x, y, or z. The dimensionless post-SN orbital separation α, the eccentricity e, and the angle θ between pre- and post-SN orbital planes can be expressed as functions of v_{kx}, v_{ky}, v_{kz}, and β:

$$\alpha = \frac{\beta}{2\beta - v_{kx}^2 - (v_{ky} + 1)^2 - v_{kz}^2} \tag{9}$$

$$1 - e^2 = \frac{v_{kz}^2 + (v_{ky} + 1)^2}{\beta^2} \left[2\beta - v_{kx}^2 - (v_{ky} + 1)^2 - v_{kz}^2 \right] \tag{10}$$

$$\cos\theta = \frac{v_{ky} + 1}{[v_{kz}^2 + (v_{ky} + 1)^2]^{1/2}}. \tag{11}$$

To obtain the distribution of binaries over post-SN characteristics we use the Jacobian transformation of the kick velocity distribution. The distribution functions for each of the three components of the kick velocity are assumed to be Gaussian:

$$p(v_{kx}, v_{ky}, v_{kz}) = \prod_j \frac{1}{\sqrt{2\pi\xi^2}} \exp\left(-\frac{v_{kj}^2}{2\xi^2}\right), \tag{12}$$

where $\xi \equiv \sigma/V_r$, and σ is the velocity dispersion of each of the one-dimensional Gaussian distributions. Using equations (9), (10), and (11) we obtain:

$$
\begin{aligned}
g(\alpha, e, \cos\theta) = & \left(\frac{\beta}{2\pi\xi^2}\right)^{3/2} \frac{2\,e}{[\alpha(1 - e^2)]^{1/2}} \left[\left(\alpha - \frac{1}{1 + e}\right)\left(\frac{1}{1 - e} - \alpha\right)\right]^{-1/2} \\
& \times \exp\left[-\frac{1}{2\xi^2}\left(\beta\,\frac{2\alpha - 1}{\alpha} + 1\right)\right] \\
& \times \exp\left[\frac{(\beta\,\alpha\,(1 - e^2))^{1/2}}{\xi^2}\cos\theta\right] \left(1 - \cos^2\theta\right)^{-1/2}.
\end{aligned}
\tag{13}
$$

In general, we are not interested in the orientation of the post-SN orbital plane. Therefore, we integrate $g(\alpha, e, \cos\theta)$ over all orientations, and obtain for the distribution

over orbital separations and eccentricities:

$$
\begin{aligned}
G(\alpha, e) \;=\;& \left(\frac{\beta}{2\pi\xi^2}\right)^{3/2} \frac{2\pi e}{[\alpha(1-e^2)]^{1/2}} \left[\left(\alpha - \frac{1}{1+e}\right)\left(\frac{1}{1-e} - \alpha\right)\right]^{-1/2} \\
& \times \exp\left[-\frac{1}{2\xi^2}\left(\beta\,\frac{2\alpha - 1}{\alpha} + 1\right)\right]\, I_o(z),
\end{aligned}
\tag{14}
$$

where

$$
z \;\equiv\; \frac{(\beta\,\alpha\,(1-e^2))^{1/2}}{\xi^2},
$$

and I_o is the modified Bessel function of zeroth order. The above expression has two singularities, at $\alpha = 1/(1 \pm e)$, which correspond to the special case of the velocity of the newborn neutron star being restricted in the y-z plane, $V_{kx} = 0$. We can see this by using equations $(9), (10)$ to obtain:

$$
v_{kx}^2 \;=\; \frac{\beta}{\alpha}\,(1-e^2)\,\left(\alpha - \frac{1}{1+e}\right)\,\left(\frac{1}{1-e} - \alpha\right)
\tag{15}
$$

In the singular cases, the distribution of kick velocities becomes two-dimensional, and there are only two independent variables describing the post-SN state: $\cos\theta$ and e (or α). The corresponding distributions are derived in Appendix B.

Scrutiny of equation (14) indicates that post-SN binaries populate only a restricted area of the $\alpha - e$ plane, independent of the orbital characteristics of the pre-SN systems. Acceptable values for α span a range from $1/(1 + e)$ to $1/(1 - e)$, limits which were first identified by Flannery & van den Heuvel (1975). Since the post-SN orbit must include the position of the two stars just prior to the explosion, the post-SN orbital separation, A_f, cannot be smaller than half of the pre-SN separation, A_i.

In addition to remaining bound, the two stars in the post-SN binary must avoid physical collision, which would probably lead to a merger. The closest distance between the two stars must at a minimum exceed the sum of their radii:

$$
A_f\,(1-e) \;>\; R_{NS} + R_2 \simeq R_2,
\tag{16}
$$

or

$$
\alpha\,(1-e) \;>\; \frac{R_2}{A_i} \equiv c.
\tag{17}
$$

This condition sets a lower limit on α, $\alpha > c/(1 - e)$, or an upper limit on e, $e < 1 - c/\alpha$. The complete set of limiting curves on the $\alpha - e$ parameter space, for a range of different values of c, is shown in Figure 2. Nevertheless, we will set $c = 0$ for simplicity in the

following discussion, returning at the very end of this section to comment on the effect of a non-zero value of c.

A two-dimensional distribution over α and e (eq. [14]) is shown in Figure 3 for the specific choice of $\beta = 0.6$ and $\xi = 1$. The behavior of the distribution is dominated by the square root term that appears in equation (14). This term becomes equal to zero along the $\alpha(1 \pm e) = 1$ curves. Variation of the values of β and ξ affects only the normalization of the distribution, and not its qualitative shape.

The distribution of post-SN systems over eccentricity, $\mathcal{J}(e)$, can be found by integrating $G(\alpha, e)$ over α. Sample distributions of post-SN systems over e are plotted in Figures 4a and 4b for different values of β and ξ. For $\beta \geq 0.5$ (Figure 4a), less than half the total mass of the pre-SN system is lost, and the binary would remain bound in the case of a symmetric explosion (no kick imparted to the neutron star). In the limit that $\xi \to 0$, the distribution over e sharply peaks at $e \to (1 - \beta)/\beta$, which is the eccentricity of the post-SN orbit if the explosion were symmetric (see, for example Verbunt 1993). As ξ increases, and the kick velocity becomes comparable to the relative orbital velocity of the stars prior to the supernova explosion, the distribution becomes broader, and then declines uniformly for $\xi \gtrsim 1$. When $\beta < 0.5$ (Figure 4b), binaries would be disrupted in the absence of any kick velocity. In the limit that $\xi \to 0$, few systems survive generally with very high eccentricities ($e \to 1$). As ξ increases, $\mathcal{J}(e)$ grows until $\xi \sim 1 - \sqrt{\beta}$, then declines. For $\xi \gg 1$, $\mathcal{J}(e)$ converges to the same asymptotic form regardless of whether $beta > 0.5$ or not:

$$\lim_{\xi \to \infty} \mathcal{J}(e) = 4\pi \left(\frac{\beta}{2\pi\xi^2} \right)^{3/2} \frac{e}{\sqrt{1 + e}} K(p), \tag{18}$$

where

$$p \equiv \sqrt{\frac{2e}{1 + e}},$$

and $K(p)$ is the complete elliptic integral with the following series representation:

$$K(p) = \frac{\pi}{2} \left\{ 1 + \left(\frac{1}{2} \right)^2 p^2 + \left(\frac{1 \cdot 3}{2 \cdot 4} \right)^2 p^4 + ... + \left[\frac{(2n - 1)!!}{2^n n!} \right]^2 p^{2n} + ... \right\},$$

where n is a positive integer.

To derive the distribution over orbital separations, $\mathcal{G}(\alpha)$, we integrate over eccentricities, e. Plots of the distribution over α are given in Figures 5a and 5b for a set of different β and ξ. When $\beta \geq 0.5$ and $\xi \to 0$ (Figure 5a), conditions are similar to those in a symmetric supernova. The distribution is narrow and peaks at $\alpha \to \beta/(2\beta - 1)$, which is the post-SN orbital separation in the absence of any kicks (see Verbunt 1993). As ξ increases

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the distribution broadens and peaks at orbital separations smaller than that before the explosion. For $\beta < 0.5$ (Figure 5b) and small kick velocities, the probability of disruption is very high and the binaries that survive have very large orbital separations. For higher values of ξ, more systems are able to reduce their energy and remain bound with smaller orbital separations. When $\xi \gg 1$, $\mathcal{G}(\alpha)$ converges to the asymptotic form:

$$\lim_{\xi \to \infty} \mathcal{G}(\alpha) = 2\pi \left(\frac{\beta}{2\pi \xi^2} \right)^{3/2} \frac{\sqrt{2\alpha - 1}}{\alpha^{5/2}}, \tag{19}$$

where $\alpha \geq 1/2$. We note that for $\xi \gg 1$, both $\mathcal{G}(\alpha)$ and $\mathcal{J}(e)$ assume forms which, apart from their normalization do not depend on either ξ nor on β.

In the preceding discussion, we have not accounted for the possibility that the two stars may collide after the supernova explosion. As a consequence of this last constraint the parameter space in α and e is further restricted, and the integrated distributions of orbital separations and eccentricities are altered (see Figures 6a and 6b). It is evident that the survival probabilities decrease dramatically if c is not very small ($c \gtrsim 0.01$), especially for large eccentricities and for orbital separations larger than that of the pre-SN binary.

2.2. Circularized Orbits

Tidal interaction between the binary members leads to circularization of the post-SN orbit, on a time scale that depends on the characteristics both of the eccentric orbit and of the companion to the neutron star. Setting aside the question of the relevant time scales we can calculate the distribution of post-SN systems over orbital separation, A_c, after circularization has been achieved.

During the circularization process, orbital energy, E, is dissipated while orbital angular momentum, J, is conserved. We define the dimensionless quantities:

$$j^2 \equiv \left(\frac{J}{J_o} \right)^2 = \alpha \, (1 - e^2), \tag{20}$$

$$\epsilon \equiv \frac{E}{E_o} = \frac{1}{\alpha}, \tag{21}$$

where $J_o^2 \equiv G \, A_i \, M_{NS}^2 M_2^2 / (M_{NS} + M_2)$ and $E_o \equiv -G \, M_{NS} M_2 / (2A_i)$. From conservation of orbital angular momentum we find for the orbital separation, A_c, of the circularized orbit:

$$j^2 = \frac{A_c}{A_i} \equiv \alpha_c. \tag{22}$$

Using equations (20), (21), and (22) we transform $G(\alpha, e)$ into the distribution of post-SN circularized systems over α_c and ϵ:

$$H(\alpha_c, \epsilon) = \pi \left(\frac{\beta}{2\pi\xi^2} \right)^{3/2} \exp\left(-\frac{2\beta+1}{2\xi^2} \right) I_o \left[\frac{\sqrt{\beta\alpha_c}}{\xi^2} \right] \frac{\exp\left(\frac{\beta}{2\xi^2}\epsilon \right)}{\sqrt{2 - \alpha_c - \epsilon}}, \tag{23}$$

$$\text{where} \quad (2 - \alpha_c - \epsilon) > 0.$$

The circularized post-SN systems are characterized by only one parameter, the orbital separation. In order to find their distribution over α_c we need to integrate $H(\alpha_c, \epsilon)$ over the dimensionless orbital energy ϵ. The limits of integration are found by considering all the constraints that viable post-SN systems must satisfy.

An upper limit to ϵ is set by the geometrical constraint, that the post-SN eccentric orbit must include the position of the stars prior to the supernova explosion (see eq. [23]),

$$\epsilon \ < \ 2 - \alpha_c. \tag{24}$$

The second constraint is that the post-SN system must be bound, and hence its orbital energy E must be negative. Since E_o has been defined to be negative, we obtain a lower limit for ϵ:

$$\epsilon > 0. \tag{25}$$

An additional lower limit is set by the need to avoid a physical collision. This condition is expressed as a lower limit on α, i.e., $\alpha > c/(1 - e)$, such that the periastron distance in the eccentric orbit exceeds the radius of the companion to the neutron star. Using equations (20), (21), and (25), we can rewrite this condition as:

$$\epsilon \ > \ \frac{2c - \alpha_c}{c^2}. \tag{26}$$

By checking for consistency ($\epsilon_{max} > \epsilon_{min}$) among the above limits, we find that from all possible values of α_c only a small range is acceptable for post-SN circularized orbits:

$$\frac{2c}{1 + c} \ < \ \alpha_c \ < \ 2. \tag{27}$$

The post-SN systems may be divided into groups depending on the value of α_c. Systems with $\alpha_c \geq 2$ become unbound (ϵ becomes negative). Systems with $\alpha_c \leq 2c/(1 + c)$ are bound, but all lead to a merger of the two stars. For $2c < \alpha_c < 2$, systems are bound and they all avoid collision. In this case, $\epsilon_{min} = 0$ and $\epsilon_{max} = 2 - \alpha_c$. Finally, systems with $2c/(1 + c) < \alpha_c < 2c$ are also bound, but a fraction of them merge. For this reason the

range of acceptable energies is further constrained: $\epsilon_{min} = (2c - \alpha_c)/c^2 > 0$. Clearly, unlike the case of a symmetric explosion, where the supernova always results in an expansion of the orbit (Verbunt 1993), the separation of the circularized orbit after an asymmetric explosion may become smaller than the pre-SN separation. However, although the lower limit on α is extended to values smaller than unity due to the kick velocity, the upper limit to the post-SN circularized orbital separation remains twice the pre-SN separation.

To obtain the distribution of post-SN binaries over α_c we integrate over ϵ. This integration can be performed analytically, yielding:

$$\mathcal{H}(\alpha_c) = \left(\frac{\beta}{2\xi^2}\right) \exp\left(\frac{-\beta\alpha_c + 1}{2\xi^2}\right) I_o\left(\frac{\sqrt{\beta\alpha_c}}{\xi^2}\right) \operatorname{erf}\left(z_o\sqrt{\frac{\beta}{2\xi^2}}\right), \qquad (28)$$

where:

$$\operatorname{erf}(x_o) \equiv \frac{2}{\sqrt{\pi}} \int_0^{x_o} e^{-x^2} \, dx,$$

$$z_o = \sqrt{2 - \alpha_c - \frac{2c - \alpha_c}{c^2}}, \qquad \frac{2c}{1 + c} < \alpha_c < 2c$$

$$= \sqrt{2 - \alpha_c}, \qquad 2c \leq \alpha_c < 2.$$

The behavior of $\mathcal{H}(\alpha_c)$ is dictated by the values of the two parameters β and ξ. Using the asymptotic forms of the error function and the modified Bessel function in the two limits that the r.m.s. kick velocity is much larger or much smaller than the relative orbital velocity in the pre-SN orbit we obtain:

$$\lim_{\xi \to 0} \mathcal{H}(\alpha_c) = \frac{1}{2\sqrt{2\pi}} \left(\frac{\beta^3}{\alpha_c}\right)^{1/4} \xi^{-1} \exp\left[-\frac{1}{2\xi^2}\left(1 - \sqrt{\beta\alpha_c}\right)^2\right], \qquad (29)$$

$$\lim_{\xi \to \infty} \mathcal{H}(\alpha_c) = \frac{z_o}{\sqrt{2\pi}} \left(\frac{\beta}{\xi^2}\right)^{3/2}. \qquad (30)$$

The distribution over α_c for different values of β and ξ is plotted in Figure 7. The behavior of $\mathcal{H}(\alpha_c)$ is analogous to that of $G(\alpha, e)$. In the limit of kicks much smaller than the relative velocity of the stars in the pre-SN orbit, conditions approximate the case of a symmetric explosion, and the distribution peaks at those values of α_c that are consistent with such an explosion: $\alpha_c = 1/\beta$ for $\beta \geq 0.5$ (Verbunt 1993), and $\alpha_c = 2$ for $\beta < 0.5$. As the average kick velocity becomes comparable to V_r, smaller orbital separations become more abundant, and for even larger kicks, a shrinkage of the orbit relative to the pre-SN state is favored.

3.   SYSTEMIC VELOCITIES

During the supernova explosion, the post-SN system as a whole receives a velocity relative to the center of mass of the pre-SN binary. We derive the probability distribution of the systemic velocities by performing a sequence of Jacobian transformations of the initial distribution of kick velocities.

We choose to work in a reference frame, in which the three axes have the same orientation as the one shown in Figure 1, but which is centered on the center of mass of the system prior to the supernova explosion. In this frame the vector velocities of the two stars are:

$$\vec{V_1} = (0, \frac{M_2}{M_1 + M_2}V_r, 0) \tag{31}$$

$$\vec{V_2} = (0, -\frac{M_1}{M_1 + M_2}V_r, 0), \tag{32}$$

where V_r is the magnitude of the relative velocity of the two stars. After the supernova explosion, $\vec{V_2}$ remains the same and $\vec{V_1}$ becomes:

$$\vec{V}_{NS} = (V_{kx}, V_{ky} + \frac{M_2}{M_1 + M_2}V_r, V_{kz}). \tag{33}$$

Hence, the systemic velocity is:

$$\begin{aligned}
\vec{V}_{sys} &= \frac{M_{NS}\vec{V}_{NS} + M_2\vec{V_2}}{M_{NS} + M_2} \\
&= \frac{1}{M_{NS} + M_2}\left(M_{NS}V_{kx}, M_{NS}V_{ky} - \frac{(M_1 - M_{NS})M_2}{M_1 + M_2}V_r, M_{NS}V_{kz}\right).
\end{aligned} \tag{34}$$

We define the dimensionless systemic velocity $v_{sys} \equiv V_{sys}/V_r$. Using equations (9), (10), and (11), we obtain:

$$v_{sys}^2 = \kappa_1 + \kappa_2 \frac{2\alpha - 1}{\alpha} - \kappa_3 \ \cos\theta \ \left[\alpha(1 - e^2)\right]^{1/2}, \tag{35}$$

where:

$$\begin{aligned}
\kappa_1 &\equiv \frac{M_1^2}{(M_1 + M_2)^2}, \\
\kappa_2 &\equiv \frac{M_{NS}^2}{(M_{NS} + M_2)(M_1 + M_2)}, \\
\kappa_3 &= 2\sqrt{\kappa_1 \ \kappa_2}.
\end{aligned} \tag{36}$$

We derived above an expression (eq. [13]) describing the distribution of post-SN systems over orbital separation, α, eccentricity, e, and orientation of the orbital plane, $\cos\theta$. Using equation (35) we can eliminate $\cos\theta$ and transform $g(\alpha, e, \cos\theta)$ into a distribution over α, e, and the magnitude of the systemic velocity, v_{sys}:

$$
\begin{aligned}
s(\alpha, e, v_{sys}) &= \left(\frac{\beta}{2\pi\xi^2}\right)^{3/2} \frac{4\,e\,v_{sys}}{\kappa_3\,\alpha\,(1-e^2)} \left[\left(\alpha - \frac{1}{1+e}\right)\left(\frac{1}{1-e} - \alpha\right)\right]^{-1/2} \\
&\times \exp\left[-\frac{1}{2\xi^2}\left(\beta\frac{2\alpha-1}{\alpha} + 1\right)\right] \exp\left[\frac{\beta^{1/2}}{\kappa_3\xi^2}\left(\kappa_1 + \kappa_2\frac{2\alpha-1}{\alpha} - v_{sys}^2\right)\right] \\
&\times \left[1 - \frac{\left(\kappa_1 + \kappa_2(2\alpha-1)/\alpha - v_{sys}^2\right)^2}{\kappa_3^2\alpha(1-e^2)}\right]^{-1/2}.
\end{aligned}
\tag{37}
$$

The above expression is valid in the general case of $v_{kx} \neq 0$ and $v_{kz} \neq 0$. The special case of $v_{kx} = 0$ has already been discussed and corresponds to the pre-SN orbital separation becoming either the periastron or the apastron distance in the post-SN eccentric orbit (see eq. [15]). In the special case of $v_{kz} = 0$, the plane of the orbit remains unaffected by the explosion, since the kick velocity is restricted in the $x - y$ plane, which is the orbital plane prior to the explosion (see eq. [11], [35] and Figure 1). The derivation of the probability density for these special cases is described in Appendix B.

It is also interesting to study how the systemic velocity imparted to the binary during the supernova explosion correlates with the orbital separation after the circularization. We transform $s(\alpha, e, v_{sys})$ to a distribution over $(\alpha_c, \epsilon, v_{sys})$ (see eq. [20], [21], [22]):

$$
\begin{aligned}
f(\alpha_c, \epsilon, v_{sys}) &= 2\left(\frac{\beta}{2\pi\xi^2}\right)^{3/2} \exp\left[\frac{1}{2\xi^2}\left(2\sqrt{\beta}\frac{\kappa_1 + 2\kappa_2}{\kappa_3} - (2\beta+1)\right)\right] \\
&\times v_{sys}\,(2 - \alpha_c - \epsilon)^{-1/2}\,\exp\left[\frac{\beta}{2\xi^2}\left(\epsilon - 2\frac{\kappa_2\epsilon + v_{sys}^2}{\kappa_3\sqrt{\beta}}\right)\right] \\
&\times \left[\kappa_3^2\alpha_c - \left(\kappa_1 + 2\kappa_2 - \kappa_2\epsilon - v_{sys}^2\right)^2\right]^{-1/2},
\end{aligned}
\tag{38}
$$

and we need to integrate over orbital energies, ϵ. The above distribution has three poles in ϵ; for clarity we rewrite it as:

$$
\begin{aligned}
f(\alpha_c, \epsilon, v_{sys}) &= \left(\frac{\beta}{2\pi\xi^2}\right)^{3/2} \frac{2}{\kappa_2}\,\exp\left[\frac{1}{2\xi^2}\left(2\sqrt{\beta}\frac{\kappa_1 + 2\kappa_2}{\kappa_3} - (2\beta+1)\right)\right] \\
&\times v_{sys}\,\exp\left[\frac{\beta}{2\xi^2}\left(\epsilon - 2\frac{\kappa_2\epsilon + v_{sys}^2}{\kappa_3\sqrt{\beta}}\right)\right] \\
&\times (\epsilon - \lambda_1)^{-1/2}\,(\lambda_2 - \epsilon)^{-1/2}\,(\lambda_3 - \epsilon)^{-1/2},
\end{aligned}
\tag{39}
$$

where

$$\lambda_1 = \frac{\kappa_1 + 2\kappa_2 - v_{sys}^2 - \kappa_3\sqrt{\alpha_c}}{\kappa_2},$$
$$\lambda_2 = \frac{\kappa_1 + 2\kappa_2 - v_{sys}^2 + \kappa_3\sqrt{\alpha_c}}{\kappa_2},$$
$$\lambda_3 = 2 - \alpha_c.$$

All three poles are numerically integrable except in the special case that:

$$\lambda_2 = \lambda_3 \Rightarrow v_{sys} = \sqrt{\kappa_1} + \sqrt{\alpha_c\kappa_2}. \tag{40}$$

In this case, the two-dimensional distribution $F(\alpha_c, v_{sys})$ becomes infinite along the line defined by equation (40). However, it is still integrable over α_c and v_{sys}, so that the final integral is finite. A sample distribution $F(\alpha_c, v_{sys})$ for a specific choice of M_1, M_2, and ξ, and for $c = 0$ is shown in Figure 8. The spikes correspond to the pole (eq. [40]). The limits on α_c are in agreement with equation (27), while the limits on v_{sys} are discussed in the next section.

We can obtain the distribution of systemic velocities only, $\mathcal{F}(v_{sys})$, by further integrating over α_c. Sample distributions normalized to the total survival fraction for different values of ξ are given in Figure 9. It is evident that the limits imposed on v_{sys} are independent of the characteristics of the kick velocity distribution. As the magnitude of the kick velocity increases the systemic velocity remains restricted to a certain range of values specified by the stellar masses. Within this range, the distribution function of v_{sys} shifts towards larger velocities as ξ increases, reaching an asymptotic distribution for $\xi \gtrsim 3$, independent of ξ (see Figure 9). As in the case of the one-dimensional distributions over eccentricities and orbital separations (see eq. [18] and [19]), this behavior is due to the fact that for $\xi \gg 1$, the exponential terms in equation (38) approach unity, and only the normalization constant depends on ξ. In this limit the normalized distribution of systemic velocities depends only on the stellar masses involved.

3.1. Limits on the systemic velocity

Scrutiny of equation (38) shows that there exists an upper and a lower limit to the values of the systemic velocities, since the expression shown is real only if

$$\kappa_3^2\alpha_c - (\kappa_1 + 2\kappa_2 - \kappa_2\epsilon - v_{sys}^2)^2 > 0, \tag{41}$$

or

$$(\kappa_1 + 2\kappa_2 - \kappa_2\epsilon - \kappa_3\sqrt{\alpha_c})^{1/2} < v_{sys} < (\kappa_1 + 2\kappa_2 - \kappa_2\epsilon + \kappa_3\sqrt{\alpha_c})^{1/2}. \tag{42}$$

In addition, we have already derived limits on ϵ and α_c (eq. [24] - [27]). By taking these into account, we can find the absolute lower and upper limits on v_{sys}:

$$
\begin{aligned}
v_{sys} &< \sqrt{\kappa_1} + \sqrt{2\kappa_2} \\
v_{sys} &> |\sqrt{\kappa_1} - \sqrt{\alpha_c \kappa_2}| > \sqrt{\kappa_1} - \sqrt{2\kappa_2}, \qquad \text{if } \frac{\kappa_1}{\kappa_2} > 2.
\end{aligned} \tag{43}
$$

The last inequality is true if $M_1 > 2\,M_{NS} = 2.8\,\mathrm{M}_\odot$ (eq. [36]), assuming a neutron star gravitational mass equal to $1.4\,\mathrm{M}_\odot$. This condition is satisfied for all the progenitors of LMXBs forming via the He-star and direct supernova mechanisms (Kalogera & Webbink 1996; Kalogera 1996), and for most of the HMXB progenitors (Portegies-Zwart & Verbunt 1996). Therefore, the maximum and minimum systemic velocities are:

$$
v_{sys}^{max} = \frac{M_1}{M_1 + M_2} + \frac{M_{NS}\sqrt{2}}{(M_{NS} + M_2)^{1/2}(M_1 + M_2)^{1/2}} \tag{44}
$$

$$
v_{sys}^{min} = \frac{M_1}{M_1 + M_2} - \frac{M_{NS}\sqrt{2}}{(M_{NS} + M_2)^{1/2}(M_1 + M_2)^{1/2}}, \qquad \text{for } M_1 > 2M_{NS} \tag{45}
$$

The above upper limit on v_{sys} is in agreement with the one found by Brandt & Podsiadlowski (1995), while the lower limit (eq. [45]) is stricter than theirs. Inspection of equation (44) shows that a maximum value of v_{sys}^{max} exists in the limit that (i) M_2 is equal to zero and (ii) M_1 is equal to the minimum possible mass of a neutron star progenitor, $\sim 2.2\,\mathrm{M}_\odot$ (e.g., Habets 1985). In this limit, the maximum of v_{sys}^{max} is ~ 2, and hence the systemic velocity of a bound post-SN binary can never exceed twice the value of the pre-SN relative orbital velocity, regardless of the magnitude of the kick velocity and the masses involved. Therefore, it becomes clear that high kick velocities do not necessarily result in high systemic velocities, as well.

Both upper and lower limits on systemic velocities can be understood physically. The maximum systemic velocity is acquired by that binary for which the neutron star receives a kick velocity oriented opposite to the pre-SN orbital velocity with a magnitude, such that its post-SN kinetic energy is just below its binding energy. The minimum systemic velocity is acquired by that binary in which the neutron star receives a kick velocity with the smallest possible magnitude needed to avoid disrupting the system due to mass loss. It is important to re-emphasize that neither the upper nor the lower limits depend on the kick velocity distribution.

4. SURVIVAL PROBABILITIES

The total survival probability of a binary system with specific initial orbital characteristics is of interest to studies of the statistical properties of an entire population

of binaries. We can obtain this survival probability by integrating over the distribution of circularized dimensionless orbital separations, $\mathcal{H}(\alpha_c)$. This integration can only be performed numerically, but it is straightforward since the function has no poles and the limits of integration are well-defined (eq. [27]).

Clearly, kick velocities to neutron stars may bind systems that would otherwise be disrupted, or disrupt those that would have remained bound. If the average kick velocity is large compared to the initial relative orbital motion of the binary components then survival depends on the small probability that the kick velocity is itself small and directed opposite to the original motion of the collapsing component. If the ratio of kick velocity to initial relative orbital velocity is small, then the survival rate of systems that would otherwise be disrupted falls very rapidly as this ratio decreases. Asymptotically, we have, respectively

$$
\begin{aligned}
\lim_{\xi\to\infty} \int_{2c/(1+c)}^{2} H(\alpha_c)d\alpha_c &= \frac{4}{3\sqrt{\pi}} \frac{(1-c)^{3/2}}{(1-c^2)} \left(\frac{\beta}{\xi^2}\right)^{3/2} \\
\lim_{\xi\to 0} \int_{2c/(1+c)}^{2} H(\alpha_c)d\alpha_c &= \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{\beta-\frac{1}{2}}{\xi\sqrt{2}}\right)\right]
\end{aligned}
\tag{46}
$$

The survival fractions for a wide range of values of ξ, and for two different values of β, are shown of Figure 10. In this illustration, ξ is varied by keeping the r.m.s. kick velocity constant, while allowing the pre-SN orbital separation to vary. Among those systems that would remain bound if collapse were symmetric ($M_1 = 3.8\,\mathrm{M_\odot}$ and $M_2 = 1.0\,\mathrm{M_\odot}$), kick velocities will tend to unbind widely separated binaries, for which the relative orbital motion falls below the kick velocity. Among systems that suffer so much mass loss in a supernova that they would otherwise be disrupted ($M_1 = 8.6\,\mathrm{M_\odot}$ and $M_2 = 1.0\,\mathrm{M_\odot}$), kick velocities will favor the survival of binary systems in just that range of separations where the relative orbital velocity is comparable to the mean kick velocity.

5.   CONCLUSION

The expressions derived here provide a tool necessary in analytical population syntheses of neutron star binaries. The additional step needed in such syntheses is to convolve the distribution over post-SN parameters with the distribution of pre-SN binaries over masses and orbital separations (see also Wijers et al. 1992). This link depends on the type of final systems and the specifics of their formation mechanism. In addition, the distribution functions of systemic velocities and their correlation with orbital separations and eccentricities (or circularized orbital separations) can be used in studying the motion

of neutron star binaries in the Galactic potential, and in modeling their spatial distribution in the Galaxy.

The results of the study presented in this paper have a number of important implications concerning the population of neutron star binaries:

There exists a correlation between orbital separations and eccentricities, which is independent of the characteristics of the binary or the magnitude of the kick velocity. For post-SN orbits much wider than the pre-SN orbit, the total energy of the binary significantly increases, and the system remains bound only in a highly eccentric orbit. On the other hand, the eccentricity may be low ($e \lesssim 0.4$) only if the post-SN orbital separation is comparable to that before the explosion. The discovery of a double neutron star system of modest eccentricity could therefore be used to infer the size of the orbit of its progenitor, provided that the gravitational radiation decay time scale for the orbit were long enough for such losses to be negligible.

The ratio of the post-SN systemic velocity, V_{sys}, to the pre-SN relative orbital velocity, V_r, is restricted in a relatively narrow range of values. Both lower and upper limits depend only on the stellar masses involved. For the ranges of progenitor masses relevant to HMXBs, LMXBs, and double neutron star binaries we find $V_{sys}^{max} \lesssim 1.5\,V_r$. Since LMXB progenitors are more tightly bound than those of HMXBs, and hence have higher relative orbital velocities than HMXB progenitors, the systemic velocities of LMXBs are expected to be higher than those of HMXBs. It is also clear that measurements of systemic velocities of neutron star binaries do not necessarily reveal information about the kick velocities imparted to neutron stars in individual systems. Instead, they can be used to infer typical relative orbital velocities prior to the supernova explosion.

Although the allowed range of systemic velocities is independent of the kick velocity, the probability distribution within this range does depend on the r.m.s. magnitude of kick velocities relative to the pre-SN orbital velocities. In the limit of very small kicks the distribution sharply peaks at values close to the lower end of the range. As the r.m.s. of the kick velocities increases the distribution becomes broader and its peak shifts to higher velocities. For kicks much higher than the pre-SN relative orbital velocities, the shape of the distribution remains unaffected, and further increases of the r.m.s. kick velocity only decrease the binary survival rate, without altering the velocity distribution of bound post-SN systems. Measurements of the systemic velocities of neutron star binaries will possibly prove quite significant in distinguishing between symmetric and asymmetric explosions with high kicks imparted to the neutron stars.

The incidence of X-ray binaries in globular clusters relates to their smallest possible

systemic velocity and to how this velocity compares with the escape velocity from the cluster. For LMXBs formed via the explosion of the He-star remnant of a common envelope phase, typical parameters for the progenitors yield $V_{sys}^{min} \simeq 100 \, \text{km s}^{-1}$ (Kalogera & Webbink 1996). The direct-SN channel (Kalogera 1996; Kalogera 1996) is fed by binaries with orbits which are much wider, but still small enough to avoid disruption by dynamical interactions. Typical parameters in this case yield $V_{sys}^{min} \simeq 20 \, \text{km s}^{-1}$. Estimates of the escape velocities from the cores of globular clusters that contain LMXBs range from $30 \, \text{km s}^{-1}$ to $60 \, \text{km s}^{-1}$ (for NGC 1851, 6440, 6441, 6624, M15, and Lil 1); more loosely bound clusters such as Ter 1 and 2 have central escape velocities of the order of $10 \, \text{km s}^{-1}$ (Webbink 1985; van Paradijs 1995). It is therefore clear that post-SN binaries formed in globular clusters from primordial binaries via the He-SN channel have a very small chance of remaining in the clusters and becoming X-ray binaries. LMXBs formed via the direct-SN channel, on the other hand, will remain in the clusters, but their formation rate is too low to account for a significant fraction of the LMXB population in globular clusters. Barring accretion-induced collapse as an alternative formation channel, it therefore appears that low-mass X-ray binaries observed in globular clusters must have formed through stellar exchanges and captures, rather than directly from primordial binaries.

We have already applied the analytical method presented here to study low-mass X-ray binaries formed via different evolutionary channels. The results of these population synthesis calculations will be presented elsewhere (Kalogera & Webbink 1996; Kalogera 1996).

Appendices

A.   Notation

The symbols of the most important physical parameters are:

M_1: mass of the exploding star,

M_2: mass of the companion to the exploding star and to the neutron star,

M_{NS}: mass of the neutron star,

R_2: radius of the companion to the neutron star,

α: ratio of the post-SN orbital separation to the pre-SN separation,

e: eccentricity of the post-SN orbit,

θ: angle between the pre-SN and post-SN orbital planes,

β: ratio of the total mass after the explosion to that before,

ξ: ratio of the standard deviation of the kick velocity distribution to the relative orbital velocity before the explosion,

c: ratio of the radius of the companion star to the pre-SN orbital separation,

α_c: ratio of the circularized orbital separation to that before the explosion,

ϵ: ratio of the post-SN binding energy to that of a system consisting of the neutron star and the companion star in an orbit with the pre-SN orbital separation,

v_{sys}: ratio of the systemic post-SN velocity to the pre-SN relative orbital velocity.

The symbols of the various distribution functions over dimensionless orbital parameters, and respective equations, in which the corresponding expressions are given, are:

$g(\alpha, e, \cos\theta)$: distribution function of post-SN orbital separations, eccentricities, and cosine of the angles between pre- and post-SN orbital planes; equation (13),

$G(\alpha, e)$: distribution function of post-SN orbital separations and eccentricities; equation (14),

$\mathcal{J}(e)$: distribution function of post-SN eccentricities for $\xi \gg 1$; equation (18),

$\mathcal{G}(\alpha)$: distribution function of post-SN orbital separations for $\xi \gg 1$; equation (19),

$H(\alpha_c, \epsilon)$: distribution function of circularized orbital separations and post-SN binding energies; equation (23),

$\mathcal{H}(\alpha_c)$: distribution function of circularized orbital separations; equation (28),

$s(\alpha, e, v_{sys})$: distribution function of post-SN orbital separations, eccentricities, and systemic velocities; equation (37),

$f(\alpha_c, \epsilon, v_{sys})$: distribution function of circularized orbital separations, post-SN binding energies, and systemic velocities; equation (38).

B. Effect of an impulse velocity

According to the numerical calculations performed by Fryxell & Arnett (1981), approximately half of the momentum carried by the ejecta intersecting the companion is transferred to it. If E_{SN} is the kinetic energy of the ejecta and V_{imp} is the velocity imparted to the companion of mass M_2, then:

$$M_2 V_{imp} \simeq \frac{1}{2}(M_1 - M_{NS}) \left(\frac{2E_{SN}}{(M_1 - M_{NS})} \right)^{\frac{1}{2}} \left(\frac{\pi R_2^2}{4\pi A_i^2} \right), \tag{B1}$$

where R_2 is the radius of the companion.

We assume that the impulse velocity, V_{imp}, is given to the companion in a direction along the line connecting the two stars, pointing away from the neutron star. In the reference frame shown in Figure 1, the velocity of the neutron star relative to that of the companion immediately after the explosion is then: $\vec{V}' = (V_{kx} + V_{imp}, V_{ky} + V_r, V_{kz})$. Following the same procedure as that described in §2, we calculate the distribution of circularized orbital separations:

$$\mathcal{H}_{imp}(\alpha_c) = \frac{\beta\pi}{(2\pi\xi^2)^{3/2}} \; \exp\left(-\frac{\beta\alpha_c + 1}{2\xi^2} \right) \; I_o\left(\frac{\sqrt{\beta\alpha_c}}{\xi^2} \right) \; S_1, \tag{B2}$$

where

$$
\begin{aligned}
S_1 &\equiv \int_{z_-}^{z_+} \exp\left(-\frac{z^2}{2\xi^2} \right) \, dz \;=\; \xi\sqrt{\frac{\pi}{2}} \left[\mathrm{erf}\left(\frac{z^+}{\xi\sqrt{2}} \right) - \mathrm{erf}\left(\frac{z^-}{\xi\sqrt{2}} \right) \right], \\
z_\pm &= v_{imp} \pm \sqrt{\beta\left(2 - \alpha_c - \frac{2c - \alpha_c}{c^2} \right)}, \qquad \text{if } \frac{2c}{1+c} < \alpha_c < 2c \\
&= v_{imp} \pm \sqrt{\beta(2 - \alpha_c)}, \qquad \text{if } 2c < \alpha_c < 2
\end{aligned}
\tag{B3}
$$

and $v_{imp} \equiv V_{imp}/V_r$, V_r being the relative orbital velocity of the pre-SN binary.

Integrating the above distribution over α_c, we find the survival probability when the impulse velocity is taken into account. In Figure 11 we plot the survival probability as a function of the pre-SN orbital separation, A_i, with and without the effect of the impulse velocity, for the specific choice of $M_1 = 4\,\mathrm{M}_\odot$, $M_2 = 1\,\mathrm{M}_\odot$ (typical of an LMXB progenitor), and $< V_k^2 >^{1/2} = 450\,\mathrm{km\,s^{-1}}$. It is clear that the survival fraction decreases due to the impulse being imparted to the companion only when the orbital separation is very small, $A_i \lesssim 3\,\mathrm{R}_\odot$. This separation is smaller than typical values of the pre-SN orbital separations of progenitors of X-ray binaries or double neutron star systems.

C. Distribution over eccentricities and orbital separations for special cases

C.1. $V_{kx} = 0$

Following the same procedure as in the general case described in § 2.1, we transform the two dimensional distribution of kicks $p_{yz}(v_{ky}, v_{kz})$ into $g'(e, \cos\theta)$, and then integrate over $\cos\theta$. The eccentricity is given by:

$$ e = \pm \left[\frac{(v_{ky} + 1)^2 + v_{kz}^2}{\beta} - 1 \right], \qquad (C1) $$

where the plus sign corresponds to $\alpha(1 - e) = 1$, the minus sign to $\alpha(1 + e) = 1$, and $\cos\theta$ is still given by equation (11). The form of the integral of $g'(e, \cos\theta)$ over $\cos\theta$ is the same as in the general case, so we obtain:

$$ \mathcal{J}'(e) = \mathcal{J}'_+(e) + \mathcal{J}'_-(e), \qquad (C2) $$

where:

$$ \mathcal{J}'_\pm(e) = \frac{\beta}{2\xi^2} \exp\left[-\frac{1}{2\xi^2} (1 + \beta(1 \pm e)) \right] I_o(z_e), $$
$$ z_e \equiv \frac{[\beta(1 \pm e)]^{1/2}}{\xi^2}. \qquad (C3) $$

The plus sign again corresponds to A_i being the periastron distance, and the minus sign to A_i being the apastron distance, in the post-SN orbit. The corresponding distribution over α is:

$$ \mathcal{G}'(\alpha) = \frac{1}{2\xi^2} \frac{\beta}{\alpha^2} \exp\left[-\frac{1}{2\xi^2} \left(1 + \beta\frac{2\alpha - 1}{\alpha} \right) \right] I_o(z_\alpha), $$
$$ z_\alpha \equiv \frac{[\beta(2\alpha - 1))/\alpha]^{1/2}}{\xi^2}. \qquad (C4) $$

C.2. $V_{kz} = 0$

In this case it is $\cos\theta = \pm 1$ (eq. [11]), that is, the post-SN orbital plane is either the same as the pre-SN one ($\theta = 0$), or it has been rotated by an angle $\theta = \pi$, and the stars orbit in a retrograde sense after the explosion. We transform the two-dimensional distribution of kicks $p_{xy}(v_{kx}, v_{ky})$ into a distribution of dimensionless orbital separations and systemic velocities, $s'(\alpha, v_{sys})$. Using equations (9), (10), and (35) we obtain the expressions relating

the four variables:

$$v_{kx}^2 = \beta\frac{2\alpha - 1}{\alpha} - \frac{\beta}{\kappa_3^2}\left(v_{sys}^2 - \kappa_1 - \kappa_2\frac{2\alpha - 1}{\alpha}\right)^2 \tag{C5}$$

$$v_{ky}^2 = \left[1 \pm \frac{\sqrt{\beta}}{\kappa_3}\left(v_{sys}^2 - \kappa_1 - \kappa_2\frac{2\alpha - 1}{\alpha}\right)\right]^2, \tag{C6}$$

where the plus sign corresponds to $\cos\theta = -1$ and the minus sign to $\cos\theta = 1$. We calculate the necessary Jacobian and find:

$$s'(\alpha, v_{sys}) = s'_+(\alpha, v_{sys}) + s'_-(\alpha, v_{sys}), \tag{C7}$$

where:

$$
\begin{aligned}
s'_{\pm}(\alpha, v_{sys}) \;=\; & \frac{2\sqrt{\beta}}{\kappa_3\pi\xi^2}\frac{v_{sys}}{\alpha^2}\left[\frac{2\alpha-1}{\alpha} - \frac{\left(v_{sys}^2 - \kappa_1 - \kappa_2\frac{2\alpha-1}{\alpha}\right)^2}{\kappa_3^2}\right]^{-1} \\
& \times \exp\left[-\frac{1}{2\xi^2}\left(1 \pm \frac{\sqrt{\beta}}{\kappa_3}\left(v_{sys}^2 - \kappa_1 - \kappa_2\frac{2\alpha-1}{\alpha}\right)\right)^2\right] \\
& \times \exp\left[-\frac{\beta}{2\xi^2}\left(\frac{2\alpha-1}{\alpha} - \frac{\left(v_{sys}^2 - \kappa_1 - \kappa_2\frac{2\alpha-1}{\alpha}\right)^2}{\kappa_3^2}\right)\right]. \tag{C8}
\end{aligned}
$$

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Figure Captions

Fig. 1— Geometry of the binary system and reference frame adopted in the calculations. The orbital plane of the pre-SN binary coincides with the plane of the page, that is the x-y plane.

Fig. 2— Limits on the parameter space $\alpha - e$ of post-SN systems for a range of values of c, the ratio of the radius of the companion to the orbital separation prior to the explosion. For point stars ($c = 0$), the allowed parameter space is restricted between the two thick lines, which correspond to limits due to the geometrical constraint $1/(1-e) > \alpha > 1/(1+e)$. The thin lines, which correspond to the constraint barring a physical collision, $\alpha > c/(1-e)$, impose a second, more stringent, lower limit to α.

Fig. 3— Distribution of post-SN systems over dimensionless orbital separations α and eccentricities e, for $\beta = 0.6$ and $\xi = 1.0$.

Fig. 4— Distribution of post-SN systems over eccentricities e for systems that (a) would remain bound ($\beta = 0.6$), or (b) be disrupted ($\beta = 0.4$), in the case of a symmetric explosion, for different values of ξ. The probability density for $\xi = 10^{-3}$ in (a) has been reduced by a factor of 100.

Fig. 5— Distribution of post-SN systems over dimensionless orbital separations for systems that (a) would remain bound ($\beta = 0.6$), or (b) be disrupted ($\beta = 0.4$), for different values of ξ.

Fig. 6— Distribution of post-SN binaries over (a) eccentricities, for $\beta = 0.6$ and $\xi = 1.0$, and over (b) dimensionless orbital separations, for $\beta = 0.6$ and $\xi = 1.0$, for different values of c.

Fig. 7— Distribution of post-SN binaries over dimensionless circularized orbital separations, α_c, for different values of β and ξ.

Fig. 8— Distribution of post-SN systems over dimensionless orbital separations of circularized orbits, α_c, and systemic velocities, v_{sys}, for $\beta = 0.6$ and $\xi = 1$.

Fig. 9— Distribution of post-SN systems over dimensionless systemic velocities, v_{sys}, for $M_1 = 3 \, M_\odot$, $M_2 = 1 \, M_\odot$ ($\beta = 0.6$), and different values of ξ.

Fig. 10— Fraction of systems that survive the supernova event, at which kick velocities with $< V_k^2 >^{1/2} = 100 \, \mathrm{km \, s^{-1}}$ are imparted to the neutron star, as a function of pre-SN orbital separation for a $1 \, M_\odot$ secondary and two different masses of the exploding star. In the case of a symmetric explosion, systems with $M_1 = 3.8 \, M_\odot$ remain marginally bound, while systems with $M_1 = 8.6 \, M_\odot$ do not survive.
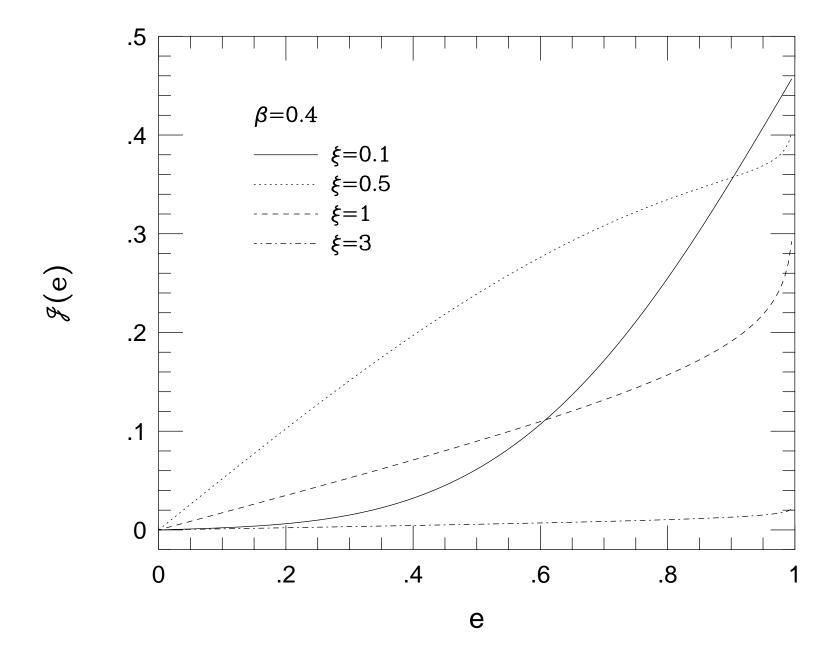
Fig. 11— Fraction of systems that survive the supernova event as a function of pre-SN orbital separation with (*dotted line*) and without (*solid line*) an impulse velocity imparted to the secondary.
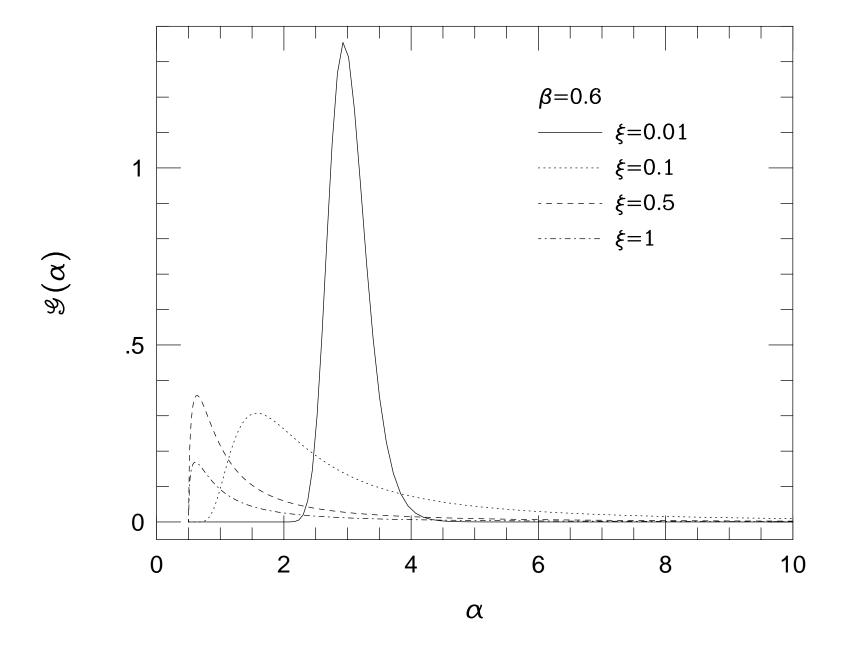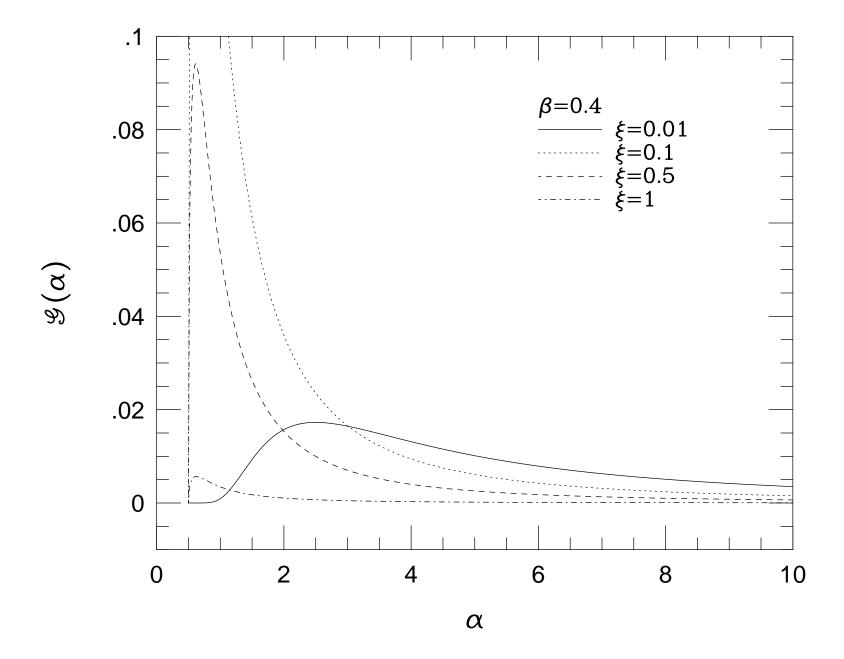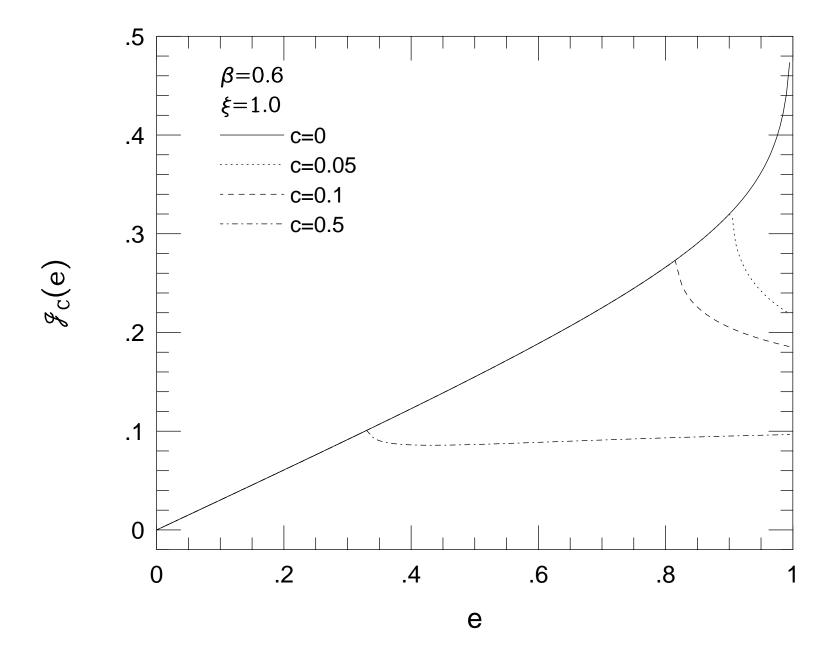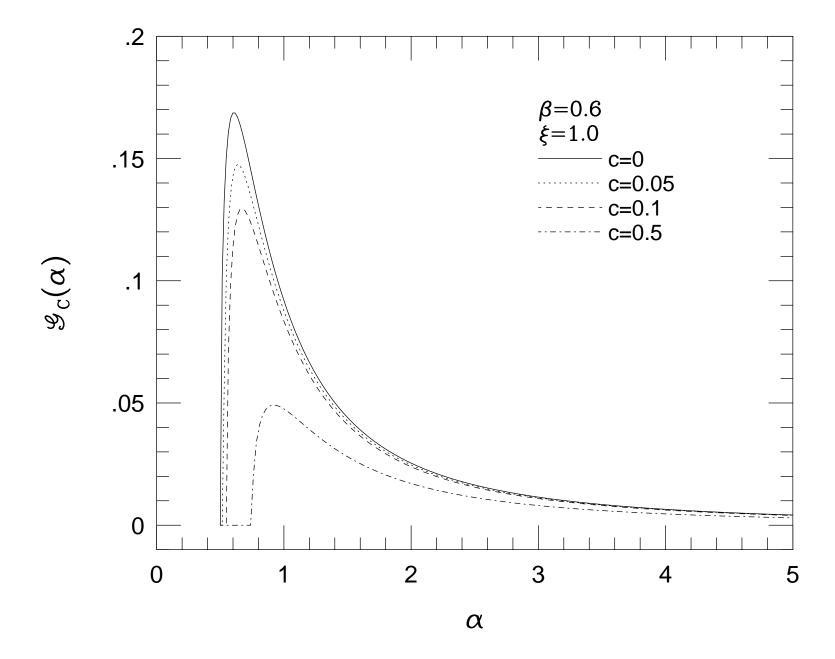
y

V_r

A_i

M_2 ● ━━━━━━━━━━━ ● M_1

x

z

$\beta = 0.6$
$\xi = 1.0$

$G(a, e)$

$\beta=0.6$

$\xi=10^{-3}$; x 10^{-2}

$\xi=0.1$

$\xi=0.5$

$\xi=1$

$\xi=3$

$\mathcal{F}(e)$

e

$\beta = 0.6$
$\xi = 1.0$

survival fraction

$\log(A_i/R_\odot)$

$M_1=8.6\ M_\odot$
$M_2=1.0\ M_\odot$

$M_1=3.8\ M_\odot$
$M_2=1.0\ M_\odot$

$\langle V_k^2\rangle^{1/2}=100\ km/s$

survival fraction vs log($A_i/R_\odot$)

$\langle V_k^2 \rangle^{1/2} = 450 \ kms^{-1}$

$M_1 = 4 \ M_\odot$

$M_2 = 1 \ M_\odot$